

ON IMPROVING EQUATION REFERENCES IN MATHEMATICAL WRITING

by

BU-625-M

August, 1977

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Abstract

A suggestion is made for improving the display of equation references in the middle of lengthy algebraic development in mathematical writing.

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The simplification of formulae in mathematical writing is usually accompanied by references back to equations developed earlier in the work. For example, suppose that in presenting sample moments we have already observed that for any constant θ

$$\sum_{i=1}^n \theta = n\theta, \quad (1)$$

and have also defined

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}. \quad (2)$$

Now we wish to develop the result

$$\sum_{i=1}^n (x_i - \bar{x})^2 = \sum_{i=1}^n x_i^2 - n\bar{x}^2.$$

We could proceed as follows:

$$\begin{aligned} \sum_{i=1}^n (x_i - \bar{x})^2 &= \sum_{i=1}^n (x_i^2 - 2\bar{x}x_i + \bar{x}^2) \\ &= \sum_{i=1}^n x_i^2 - 2\bar{x} \sum_{i=1}^n x_i + \sum_{i=1}^n \bar{x}^2. \end{aligned} \quad (3a)$$

Because of (1) and (2), equation (3a) becomes

$$\begin{aligned}
 \sum_{i=1}^n (x_i - \bar{x})^2 &= \sum_{i=1}^n x_i^2 - 2\bar{x}\sum_{i=1}^n x_i + n\bar{x}^2 \\
 &= \sum_{i=1}^n x_i^2 - 2n\bar{x}^2 + n\bar{x}^2 \\
 &= \sum_{i=1}^n x_i^2 - n\bar{x}^2 .
 \end{aligned} \tag{4a}$$

Notice that between (3a) and (4a) there is an interruption to the flow of the algebra: (3a) gets simplified by using (1) and (2), and to draw attention to this the development is temporarily halted at (3a), and a new sentence used. Then, to pick up the thread of the algebra again, the left-hand side of (3a) must be written down again, in (4a).

An alternative development is the following:

$$\begin{aligned}
 \sum_{i=1}^n (x_i - \bar{x})^2 &= \sum_{i=1}^n (x_i^2 - 2\bar{x}x_i + \bar{x}^2) \\
 &= \sum_{i=1}^n x_i^2 - 2\bar{x} \sum_{i=1}^n x_i + \sum_{i=1}^n \bar{x}^2
 \end{aligned} \tag{3b}$$

$$= \sum_{i=1}^n x_i^2 - 2\bar{x}n\bar{x} + n\bar{x}^2, \text{ from using (1) and (2),} \tag{4b}$$

$$= \sum_{i=1}^n x_i^2 - 2n\bar{x}^2 + n\bar{x}^2$$

$$= \sum_{i=1}^n x_i^2 - n\bar{x}^2 .$$

Here there is no interruption between (3b) and (4b) and the flow of the algebra is an unbroken series of steps from one expression to another — for many readers,

easier to read than the development using (3a) and (4a). However, in (4b) there is now the phrase "from using (1) and (2)" following the algebra. It achieves the same thing as the interruptive sentence used between (3a) and (4a), of giving reasons for the simplifications that are used. However, although grammatically satisfactory, this phrase is logically in a most unsatisfactory position. It indicates how the step is made from (3b) to (4b), but it comes after the algebra in (4b). Logically, it should come before (4b). But grammatically, the only way we seem able to achieve this is by some kind of interruptive sentence as illustrated between (3a) and (4a).

In a multitude of styles, both of these ways of referring back to already-stated equations are used frequently in mathematical writing. And yet both of them have deficiencies insofar as easy readability is concerned, particularly for students, for whom easy reading of algebra should surely be an important feature of the written material they use - textbooks and class notes. The first method, the interruptive sentence, interrupts the flow of the algebra and the second method, the tagged-on phrase, is logically unsatisfying. Furthermore, both methods require the reader to look back in his reading to where the referred-to equations are located.

These deficiencies are greatly exacerbated when the algebra is more complicated than in the illustration used here. This is especially so when references back are more numerous than just two, and when they are several pages, or maybe chapters, earlier in the work concerned. Readers must sometimes wish they had more than five fingers on their left hands to keep a book open at the many pages sometimes referred to.

The problem is to maintain the flow of mathematical development with logically placed references but without interruptive sentences or tagged-on phrases. Gram-

matically, this presents an inconsistency: to put the references before (4a) we can only end the sentence after (3a), or to put them after (4a) we can only use tagged-on phrases. To avoid both these unsatisfying styles we need a new style convention - and surely one can be adapted from the long tradition of having parenthetical numbers to label equations, without their playing any part at all in the grammar of the algebraic sentence such as that, for example, of which (3b) and (4b) are a part. Extension of this tradition leads to suggesting the use of parenthetical phrases or sentences for references. For example:

$$\begin{aligned} \sum_{i=1}^n (x_i - \bar{x})^2 &= \sum_{i=1}^n (x_i^2 - 2\bar{x}x_i + \bar{x}^2) \\ &= \sum_{i=1}^n x_i^2 - 2\bar{x} \sum_{i=1}^n x_i + \sum_{i=1}^n \bar{x}^2 \end{aligned} \quad (3c)$$

$$[\text{Use (2): } \sum_{i=1}^n x_i = n\bar{x} .]$$

$$[\text{Use (1): } \sum_{i=1}^n \bar{x}^2 = n\bar{x}^2 .]$$

$$= \sum_{i=1}^n x_i^2 - 2\bar{x}n\bar{x} + n\bar{x}^2 \quad (4c)$$

$$= \sum_{i=1}^n x_i^2 - n\bar{x}^2 .$$

The sentence contained in the square brackets between (3c) and (4c) are, of themselves, grammatically correct. For using the information they contain, they come logically in the correct position, and although they create a break in the spacing of the main development it is only a spatial interruption, not a linguistic one, with no need for repeating the left-hand side of the main development, as in (4a). Certainly, these parenthetical statements are grammatically wrong inasmuch

as they come in the middle of a sentence — and in some mathematical developments there may be several uses of such statements. But this is a convention of style that mathematical writing could profit from — to accept these sentences for what they are: reminders to the reader of earlier results that need to be used.

Positioning them well to the right of the main line of equality symbols not only makes the square brackets themselves of no real practical necessity, but also draws immediate attention to their not being grammatically part of the main development — but mathematically they form an integral part of the reader's easy understanding of that development. Furthermore, this style can in many cases, as here, give not only the equation number of the equation being referred to but also the equation itself or some "obvious" consequence of it. Not only does this reduce the reader's need for a many-fingered left hand, but some repetition of important results in these parenthetical sentences throughout a long piece of writing would surely help the reader to learn those results.

Can a convention like this become adopted in mathematical writing? Is it worthwhile? (Indications are that students like it.) Is there something already available? What better ideas can be engendered?